able correlation for supersonic conditions is obtained utilizing expression (2) for the two nozzle shapes studied.

Reference

[1] Lewis, C. H. and Carlson, D. J., "Normal shock location in underexpanded gas and gas-particle jets," AIAA J. **2**, 776–777 (1964).

[2] D'Attorre, L. and Harshbarger, F., "Experimental and theoretical studies of underexpanded jets near the Mach disc," General Dynamics/Astronautics Rept. GDA-DBE-64-008 (February 1964).

[3] D'Attorre, L. and Harshbarger, F., "Further experimental and theoretical studies of underexpanded jets near the Mach disc," General Dynamics/Astronautics Rept. GDA/DBE-64-041 (July 1964).

[4] D'Attorre, L. and Harshbarger, F., "The behavior of the Mach disc up to moderate pressure ratios," General Dynamics/Astronautics Rept. GD-DBE-64-042 (July 1964).

[5] Foelsch, K., "The analytical design of an axially symmetric laval nozzle for a parallel and uniform jet," J. Aeronaut. Sci. **16**, 161–166 (1949).

[6] Bowyer, J., D'Attorre, L., and Yoshihara, H., "Transonic aspects of hypervelocity rocket plumes," *Supersonic Flow, Chemical Processes and Radiative Transfer*, edited by D. B. Olfe and V. Zakkay (Pergamon Press, Ltd., London, 1964), pp. 203–209.

[7] Love, E. S., Grigsby, E. C., Lee, L. P., and Woodling, M. J., "Experimental and theoretical studies of axisymmetric free jets," NASA TR R-6 (1959).

Integrated Solar Pressure for a Transparent Balloon Satellite

Francis E. Baird*
The Boeing Company, Seattle, Wash.

A BALLOON satellite is subject to an orbit perturbing force due to solar radiation pressure which is greater than the atmospheric drag force for medium- and high-altitude orbits. To minimize this perturbation, certain space experiments may require a transparent balloon that allows most of the solar radiation to pass through it. A method for calculating this force exerted on a transparent balloon satellite in terms of the index of refraction is given below. This method accounts for multiple reflections at the surface of the balloon satellite. Results of calculation are presented at the end of this paper.

Figure 1 shows the passage of a light ray through a balloon. Part of the ray is reflected at each interface and exerts a force that has a component parallel to the incident ray. This component, integrated over the surface, is the perturbing force. At each of the points A, B, C, etc. of Fig. 1, the ray breaks up into an infinite number of transmitted and reflected rays as shown by Fig. 2. The fraction reflected at the first surface is a function of the incidence angle i between the ray and the normal to the surface and is given by[1]

$$r = \frac{1}{2}\left[\frac{\sin^2(i-t)}{\sin^2(i+t)} + \frac{\tan^2(i-t)}{\tan^2(i+t)}\right] \qquad (1)$$

where r is the fraction reflected, i the incidence angle between ray and normal, and t is the transmittance angle between ray and normal.

The incidence angle i is related to the transmittance angle t by Snell's law:

$$\sin i = n \sin t$$

where n is the refractive index of the balloon material.

Received August 26, 1964.

* Research Engineer, Aerospace Division.

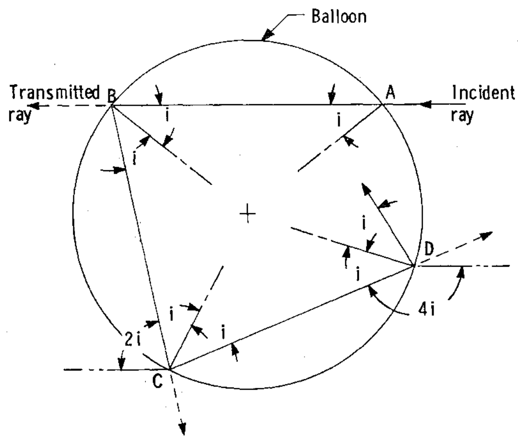

Fig. 1 Path of a light ray through a transparent spherical balloon satellite.

Since both the incident and reflected rays exert a component in the direction of the incident ray,[2] it is evident from the geometry of Fig. 2 that at each of the points a through d we have:

At point a:

Fraction reflected: r
Fraction transmitted: $1 - r$
Component parallel to incident ray: $r\,(1 + \cos 2i)$

At point b:

Fraction reflected: $r(1 - r)$
Fraction transmitted: $(1 - r)^2$
Component parallel to incident ray: $r(1 - r)[\cos(i - t) + \cos(i + t)]$

At point c:

Fraction reflected: $r^2(1 - r)$
Fraction transmitted: $r(1 - r)^2$
Component parallel to incident ray: $-r^2(1 - r)[\cos(i - t) + \cos(i + t)]$

At point d:

Fraction reflected: $r^3(1 - r)$
Fraction transmitted: $r^2(1 - r)^2$
Component parallel to incident ray: $r^3(1 - r)[\cos(i - t) + \cos(i + t)]$

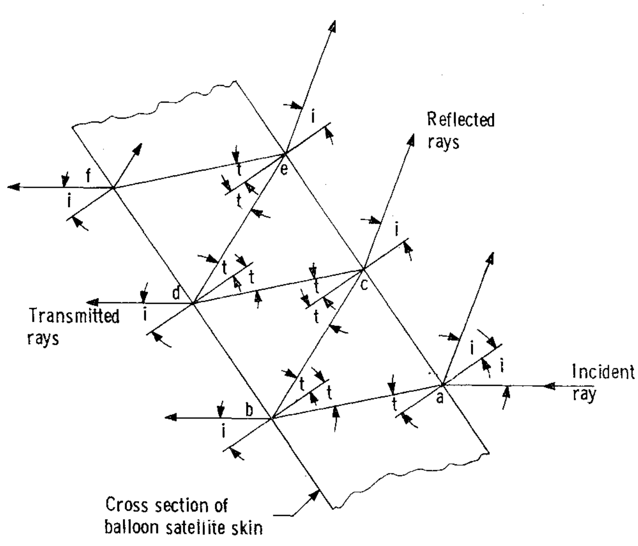

Fig. 2 Gemoetry of multiple reflection for a single incident ray.

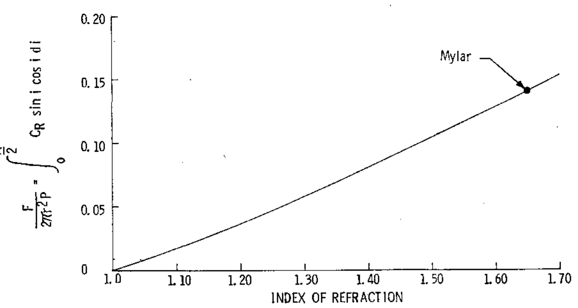

Fig. 3 Integral from Eq. (6) plotted against index of refraction.

Summation of the fractions gives: the amount reflected (R),

$$R = r + (1 - r)^2(r + r^3 + r^5 + \ldots)$$
$$= 2 - 2 \sum_{n=0}^{\infty} (-r)^n = \frac{2r}{1+r} \tag{2}$$

the amount transmitted (T),

$$T = (1 - r)^2 (1 + r^2 + r^4 + r^6 + \ldots)$$
$$= -1 + 2 \sum_{n=0}^{\infty} (-r)^n = \frac{1-r}{1+r} \tag{3}$$

and the component parallel to incident ray (C),

$$C = r(1 + \cos 2i) + [\cos(i - t) + \cos(i + t)](1 - r)(r - r^2 + r^3 - r^4 + \ldots)$$
$$= 2r \cos^2 i + 2 \cos i \cos t\, r(1 - r) \sum_{n=0}^{\infty} (-r)^n \tag{4}$$
$$= 2r \cos i\, (\cos i + [(1 - r)/(1 + r)] \cos t)$$

At point B of Fig. 1 the angles of incidence and transmittance are the same as at point A if the balloon is spherical, but the amount of light is smaller and equal to that transmitted through point A. The component parallel to the incident ray is CT. At point C, the incident light is that reflected from point B and its component parallel to the original incident ray at point A is $-CTR \cos 2i$. Similar reasoning at subsequent points yields the following:

$$C_R = C + CT\,(1 - R \cos 2i + R^2 \cos 4i - R^3 \cos 6i + \ldots) \tag{5}$$

where C_R is the total reflected component from a ray incident at point A of Fig. 1.

If the ray striking point A is considered to be a pencil of rays of cross-sectional area $dA \cos i$ striking an area dA on the balloon, the total force on the balloon F in the direction of the incident ray is

$$F = 2\pi r^2 P \int_0^{\pi/2} C_R \sin i \cos i \; d\,i \tag{6}$$

where P is the solar pressure 9.40×10^{-8} lb/ft^2, and r is the radius of the balloon satellite. This is the final equation. Figure 3 shows the integral plotted against the index of refraction for a range of refractive indices covering most transparent plastics.

References

[1] Born, M. and Wolf, E., *Principles of Optics* (Pergamon Press, New York, 1959), Chap. I, p. 44.

[2] Strong, J., *Concepts of Classical Optics* (W. H. Freeman and Co., Inc., San Francisco, 1958), Chap. IV, p. 75.

Correlation of Two-Dimensional and Axisymmetric Boundary-Layer Flows for Purely Viscous Non-Newtonian Fluids

Nisiki Hayasi*
NASA Ames Research Center, Moffett Field, Calif.

For steady flows in Newtonian fluids, Mangler,[1] Stepanov,[2] and Hatanaka[3] independently showed that the equations for the axisymmetric laminar boundary layer can be reduced to the equations for the two-dimensional laminar boundary layer by the transformations

$$\bar{x} = \int_0^x r^2(x)dx \qquad \bar{y} = r(x)y$$
$$r(x)\bar{v} = v + \frac{1}{r^2}\frac{dr}{dx}\bar{y}u \tag{1}$$

where x is the distance along the wall from the forward stagnation point, y is the distance from the wall, u and v are components of the velocity in the directions of x and y, respectively, and r is the distance from the axis to the wall.

The purpose of the present note is to show that, for steady flows in purely viscous non-Newtonian fluids, the equations for the axisymmetric laminar boundary layer also can be reduced to those for the two-dimensional laminar boundary layer by generalized transformations.

The equations for the steady axisymmetric boundary layer for purely viscous incompressible non-Newtonian fluids can be written as

$$(\partial/\partial x)(ru) + (\partial/\partial y)(rv) = 0 \tag{2}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{dp}{dx} + \frac{\partial \tau_{yx}}{\partial y} \tag{3}$$

where ρ is the density, p the pressure, and τ_{yx} is the shear stress in the x direction because of the velocity gradient in the y direction. From Bernoulli's theorem, we have

$$-(1/\rho)(dp/dx) = U(dU/dx) \tag{4}$$

It has been shown by Schowalter[4] that, for two-dimensional flows, shear stress can be expressed as

$$\tau_{yx} = a\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y} \qquad (n > 0) \tag{5}$$

where a and n are constants, provided that the Ostwald-de Waele (power-law) model is chosen. It can be shown easily that this equation is also valid for axisymmetric flows. Putting Eqs. (4) and (5) into Eq. (3), we have

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + \nu\frac{\partial}{\partial y}\left\{\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right\} \tag{6}$$

where $\nu = a/\rho$.

After the transformations

$$\bar{x} = \int_0^x r^{n+1}\,dx \qquad \bar{y} = ry$$
$$r^n\bar{v} = v + \frac{1}{r^2}\frac{dr}{dx}\bar{y}u \tag{7}$$

Eqs. (2) and (6) are reduced to

$$\partial u/\partial\bar{x} + \partial\bar{v}/\partial\bar{y} = 0 \tag{8}$$

$$u\frac{\partial u}{\partial \bar{x}} + \bar{v}\frac{\partial u}{\partial \bar{y}} = U\frac{dU}{d\bar{x}} + \nu\frac{\partial}{\partial \bar{y}}\left\{\left|\frac{\partial u}{\partial \bar{y}}\right|^{n-1}\frac{\partial u}{\partial \bar{y}}\right\} \tag{9}$$

Received August 28, 1964.

* Research Associate, National Academy of Sciences, National Research Council. Member AIAA.